$$f(t; A, b, k) = A e^{bt} + k$$

$$r(t_i, c_i ; A, b, k) = c_i - \hat{c}_i$$

$$= c_i - f(t_i)$$

$$= c_i - A e^{bt_i} - k$$

$$p(t_i, c_i ; A, b, k) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(c_i - Ae^{bt_i} - k)^2}$$

$$J(\vec{t}, \vec{c} ; A, b, k) = \prod_{i=1}^{50} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(c_i - Ae^{bt_i} - k)^2}$$

$$L(A, b, k ; \vec{t}, \vec{c}) = \prod_{i=1}^{50} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(c_i - Ae^{bt_i} - k)^2}$$

$$\ell(A, b, k ; \vec{t}, \vec{c}) = \ln\left(\prod_{i=1}^{50} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(c_i - Ae^{bt_i} - k)^2} \right)$$

$$= \sum_{i=1}^{50} \ln\left(\frac{1}{\sqrt{2\pi}}\right) + \ln\left(e^{-\frac{1}{2}(c_i - Ae^{bt_i} - k)^2}\right)$$

$$= \sum_{i=1}^{50} \ln\left(\frac{1}{\sqrt{2\pi}}\right) + \sum_{i=1}^{50} -\frac{1}{2}(c_i - Ae^{bt_i} - k)^2$$

$$= 50 \ln\left(\frac{1}{\sqrt{2\pi}}\right) + \sum_{i=1}^{50} -\frac{1}{2}(c_i - Ae^{bt_i} - k)^2$$

$$\frac{\partial \ell}{\partial A} = \frac{\partial}{\partial A}\left(50 \ln\left(\frac{1}{\sqrt{2\pi}}\right)\right) + \sum_{i=1}^{50} \frac{\partial}{\partial A}\left(-\frac{1}{2}(c_i - Ae^{bt_i} - k)^2\right)$$

$$= 0 + \sum_{i=1}^{50} -\frac{1}{2}\frac{\partial}{\partial A}\left((c_i - Ae^{bt_i} - k)^2\right)$$

$$= \sum_{i=1}^{50} -\frac{1}{2}\left(2(c_i - Ae^{bt_i} - k)(-e^{bt_i})\right)$$

$$\boxed{\begin{array}{ll} h(u) = u^2 & u = c_i - Ae^{bt_i} - k \\ \frac{dh}{du} = 2u & \frac{\partial u}{\partial A} = 0 - e^{bt_i}\frac{\partial A}{\partial A} - 0 \\ & = -e^{bt_i}(1) \\ & = -e^{bt_i} \end{array}}$$

$$= \sum_{i=1}^{50} (c_i - Ae^{bt_i} - k)(e^{bt_i})$$

$$\frac{\partial \ell}{\partial b} = \frac{\partial}{\partial b}\left(50 \ln\left(\frac{1}{\sqrt{2\pi}}\right)\right) + \sum_{i=1}^{50} \frac{\partial}{\partial b}\left(-\frac{1}{2}(c_i - Ae^{bt_i} - k)^2\right)$$

$$= 0 + \sum_{i=1}^{50} -\frac{1}{2}\frac{\partial}{\partial b}\left((c_i - Ae^{bt_i} - k)^2\right)$$

$$= \sum_{i=1}^{50} -\frac{1}{2}\left(2(c_i - Ae^{bt_i} - k)(0 - Ae^{bt_i}(t_i) - 0)\right)$$

$$= \sum_{i=1}^{50} (c_i - Ae^{bt_i} - k)(At_i e^{bt_i})$$

$$\frac{\partial \ell}{\partial k} = \frac{\partial}{\partial k}\left(50 \ln\left(\frac{1}{\sqrt{2\pi}}\right)\right) + \sum_{i=1}^{50} \frac{\partial}{\partial k}\left(-\frac{1}{2}(c_i - Ae^{bt_i} - k)^2\right)$$

$$= 0 + \sum_{i=1}^{50} -\frac{1}{2}\frac{\partial}{\partial k}\left((c_i - Ae^{bt_i} - k)^2\right)$$

$$= \sum_{i=1}^{50} -\frac{1}{2}\left(2(c_i - Ae^{bt_i} - k)(-1)\right)$$

$$= \sum_{i=1}^{50} c_i - Ae^{bt_i} - k$$